UAB-FT-324

The Neutrino Mass Spectrum in the Supersymmetric Flipped SU(5)⊗U(1) GUT Model

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Abstract

In the context of the supersymmetric flipped $SU(5) \otimes U(1)$ GUT model, we have studied in detail the neutrino mass spectrum, obtaining an approximate formula for the corresponding effective three-generation mass matrix. It is found that at least two neutrinos are expected to have extremely small masses of no particular interest. The third neutrino, on the other hand, may eventually get a mass of order 10^{-3} eV, of relevance for the solution of the solar neutrino problem with the MSW mechanism, as a consequence of vacuum-expectation-values at the GUT mass scale for some of the scalar partners of the right-handed neutrinos.

November 1993

In the last few years there has been a considerable activity in the area of the GUT models derivable from superstring theories. Among the others, the most celebrated is perhaps the so-called "flipped" $SU(5) \otimes U(1)$ model, in its minimal supersymmetric N=1 GUT version [1]. It has several nice features, which make it one of the most promising GUT models which have been constructed so far. First of all, it does not require the presence of the adjoint Higgs representation (not allowed in models based on string theories with Kac-Moody level K=1) for obtaining the correct spontaneous symmetry breaking down to the standard model. Furthermore, it produces automatically the so-called doublet-triplet mass splitting, essential for avoiding an unplausible fast proton decay mediated by the exchange of light color-triplet scalars. There are also extended "string"-type versions of this model [1,2], which have been extensively studied in the literature. However, most of them reduce at low-energy to the minimal version we are considering in the present paper. In view of its successes, it has become of primary importance the study of its consequences and predictions, and in particular its expectations for the physics of fermion masses. As far as the charged sector is concerned, in the *flipped* model the quark and the charged-lepton masses turn out to be uncorrelated (unless one embeds the model in $SO(10)$), so that one does not expect any constraint from the observed mass spectra. On the other hand, the model mantains the equality (at the GUT mass scale, M_G) of the up-quark and the Dirac neutrino mass matrices, implying therefore the need of a seesaw-type of mechanism for the suppression of the neutrino masses [12]. The study of the possible seesaw scenarios which may be implemented in the model, both in the supersymmetric and in the non-supersymmetric case, has been given in refs.[3,4]. Also the present paper is devoted to the discussion of the possible neutrino mass spectra which may arise in the flipped model, but here we shall be able to carry out a more detailed study, at a three-generation level, and we shall derive a general formula for the corresponding mass matrix, holding also in presence of non-vanishing vacuum-expectation-values (VEVs) for the scalar partners of the right-handed (RH) neutrinos.

The introduction of these VEVs, $< \nu^c >$, was motivated in ref.[5], by the study of the mass relations between the d-type quarks and the charged leptons (not holding in the flipped case) which one obtains in the context of the $SU(4) \otimes O(4)$ model[1]. As it is

[1] We recall that, as shown in refs.[3,4], the $SU(4) \otimes O(4)$ and the flipped $SU(5) \otimes U(1)$

well known, these relations, also valid in the standard $SU(5)$ GUT model, are consistent with the actual masses only for the third generation, corresponding to the famous relation (at a low-energy scale) $m_b \simeq 3m_\tau$, where the factor of 3 arises from the different mass renormalization in the two fermion sectors. Leaving aside the problem with the masses of the first generation, in view of possible large effects of non-perturbative QCD on the light d-quark mass, one may expect a better success for the second generation, since m_s is larger (or of the same order) than Λ_{QCD}. A way for preserving the successful relation between m_b and m_τ, while improving the one involving m_s and m_μ, was suggested in ref.[5], by assuming non-vanishing VEVs for the RH sneutrinos. In particular, it was found that in order to fit m_s and m_μ to their actual values, $< \nu^c_\mu >$ had to be set at the GUT mass scale, M_G. The general consequences of this result on the structure of the neutrino mass spectrum were already studied in ref.[4], where it was shown that in this case sizeable (*i.e.,* non-negligibly small) masses could be obtained without introducing non-renormalizable terms.

Before proceeding to the discussion of the neutrino sector, let us briefly review the main ingredients of the flipped $SU(5) \otimes U(1)$ supersymmetric GUT model. Its superfield content has been given in the Table of ref.[3], to which we also refer for our notations. The matter superfields, which accomodate for each generation the fifteen fermions of the Standard Model (SM) plus the right-handed neutrino ν^c, form a 16-dimensional spinorial representation of $SO(10)$, $\mathbf{F}(\mathbf{10},\mathbf{1}) \oplus \bar{\mathbf{f}}(\bar{\mathbf{5}},-\mathbf{3}) \oplus \mathbf{l^c}(\mathbf{1},\mathbf{5})$. The particles fit in these supermultiplets as in the Standard $SU(5)$ model (to which is added the RH neutrino), but with the exchange $u^{(c)} \leftrightarrow d^{(c)}$, and $\nu^{(c)} \leftrightarrow e^{(c)}$, which is the reason for the term "flipped". Therefore, we have $\mathbf{F} \sim (u,d,d^c,\nu^c)$, $\mathbf{f} \sim (u^c,\nu,e)$ and $\mathbf{l^c} \sim e^c$. The gauge group of the model is spontaneously broken down to the SM at the GUT mass scale, $M_G = 10^{16}$ GeV, by the VEV of the neutral components of two Higgs superfields $\mathbf{H}(\mathbf{10},\mathbf{1})$ and $\bar{\mathbf{H}}(\bar{\mathbf{10}},-\mathbf{1})$, $< H > \equiv < \nu^c_H >$ and $< \bar{H} > \equiv < \bar{\nu}^c_H >$. From the minimization of the F and the D terms one finds that these two VEVs must be equal, so that we may set $< \nu^c_H > = < \bar{\nu}^c_H > \equiv M_G$. Through this spontaneous symmetry breaking the super-heavy gauge bosons and their corresponding super-partners, the gauginos, get a mass of order M_G by "absorbing" the would-be Goldstone bosons u^c_H, $\bar{u}^c_H$, e^c_H, $\bar{e}^c_H$, and a linear combination[2] of ν^c_H and $\bar{\nu}^c_H$.

models give in general very similar results in the neutral lepton sector.

[2] The corresponding orthogonal combination of ν^c_H and $\bar{\nu}^c_H$ remains "uneaten", but still

Moreover, also the fermionic components of $\mathbf{H}$ and $\bar{\mathbf{H}}$, and in particular of ν^c_H and $\bar{\nu}^c_H$, forming massive states with the heavy gauginos, get a large mass at the GUT scale.

The subsequent electroweak breaking down to $SU(3)_c \otimes U(1)_{em}$ is induced spontaneously by the VEVs of two Higgs superfields $\mathbf{h}(\mathbf{5}, -\mathbf{2}) \sim (D_3; h^o, h^-)$ and $\bar{\mathbf{h}}(\bar{\mathbf{5}}, \mathbf{2}) \sim (\bar{D}_3; \bar{h}^+, \bar{h}^o)$, which form a 10-dimensional representation of $SO(10)$; we shall set $< h >=< h^o >\equiv v_d$ and $< \bar{h} >=< \bar{h}^o >\equiv v_u$, such that $(v_u^2 + v_d^2)^{1/2} = v_{SM} = 246$ GeV. One of the nice features of the model, as we have already mentioned above, is the natural solution of the so-called doublet-triplet mass-splitting problem. This is related to the fact that if the colour-triplet scalar fields D_3 and $\bar{D}_3$ have a mass of order M_W (like their $SU(2)_L$-doublet partners (h^o, h^-) and $(\bar{h}^+, \bar{h}^o)$), they would mediate a too fast proton decay, in contradiction with the present experimental bounds. In order to solve this problem, also present in the standard GUTs, one needs to find a mechanism for inducing a large mass of order M_G to these colour-triplets, while leaving unaffected the mass of the standard-type Higgs doublets. A mechanism of this sort is naturally implemented in the flipped model since D_3 and $\bar{D}_3$ form massive states at the GUT scale with the (uneaten) colour-triplets d^c_H and $\bar{d}^c_H$ of the Higgs superfields $\mathbf{H}$ and $\bar{\mathbf{H}}$.

The superfield content of the model is then completed by a set of n_g+1 $SU(5)$-singlets ϕ_α ($n_g = 3$ is the number of generations) which may naturally arise in string theories, and which, mixing with the RH neutrinos ν^c, are essential for providing the desired seesaw mechanism. The scalar component of one of these singlets, denoted by ϕ_o, develops a VEV at the weak scale. This, apart from giving a mass to all ϕ's, is also necessary for producing the correct mixing between the two electroweak Higgs doublets, insuring the non-vanishing of both v_u and v_d, and avoiding therefore the presence of an unacceptable electroweak axion.

Finally, the model is completely specified by writing the most general superpotential, which has to satisfy the discrete Z_2 symmetry $\mathbf{H} \to -\mathbf{H}$ in order to prevent unplausible tree-level Majorana masses (of order M_G) for the ordinary left-handed (LH) neutrinos:

$$\begin{aligned}\mathcal{W}_{(5)} =& \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i l^c_j h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\ &+ \lambda_6^{i\alpha} F_i \bar{H} \phi_\alpha + \lambda_7^{\alpha} h \bar{h} \phi_\alpha + \lambda_8^{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \,.\end{aligned} \tag{1}$$

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gets a mass of order M_G.

Notice that this superpotential contains all possible terms which satisfy the $SO(10)$ symmetry. After the electroweak symmetry breaking the first three terms generate masses for all fermions, including Dirac masses for the neutrinos. More precisely, the corresponding mass matrices may be written as follows:

$$m_d = \lambda_1 \, < h^o > \equiv \lambda_1 \, v_d \,, \tag{2a}$$

$$m_u = m_{\nu D} = \lambda_2 \, < \bar{h}^o > \equiv \lambda_2 \, v_u \,, \tag{2b}$$

$$m_e = \lambda_3 \, < h^o > \equiv \lambda_3 \, v_d \,, \tag{2c}$$

where the λ_i's (i=1,2,3) are the 3×3 matrices of the Yukawa-type couplings, and $m_{\nu D}$ is the Dirac mass matrix for the neutrinos. Notice the lack of any mass correlation among the different charged fermion sectors, and in particular of the Standard-$SU(5)$ relations $m_{d_i}(M_G) = m_{e_i}(M_G)$. The λ_4- and the λ_5-terms give rise to the necessary doublet-triplet mass-splitting discussed above, producing mass terms of the type $\lambda_4 M_G(D_3 d^c_H)$ and $\lambda_5 M_G(\bar{D}_3 \bar{d}^c_H)$. The λ_6-term produces the mixing between the singlets[3] ϕ_α (α=1,2,3) and the RH neutrinos ν^c_i. The λ_7-term gives rise to the correct mixing between the two electroweak Higgs doublets, and the last λ_8-term, which yields mass terms for the ϕ's of order M_W, is necessary for preventing them from developing large VEVs. Furthermore, when also the scalar partners of the RH neutrinos have a non-vanishing VEV, $< \nu^c_i >$, the λ_6-term leads to a mixing between the singlets ϕ_α and the heavy state $\bar{\nu}^c_H$:

$$\sum_{i=1}^{3} \lambda_6^{i\alpha} < \nu^c_i > \; (\phi_\alpha \, \bar{\nu}^c_H) \,, \tag{3}$$

which will play a primary role in our discussion of the neutrino mass matrix.

The study of the possible neutrino mass spectra which may arise in the context of the flipped $SU(5) \otimes U(1)$ model has already been carried out by several authors [3,4,6,7]. Here is a brief account of the results obtained more recently[4]. At first , it was thought

[3] As in the previous papers [3,4], we assume for simplicity that the fermionic partner of the singlet ϕ^o does not mix with the ν^c_i.

[4] The Feynman diagrams which explain the origin of neutrino masses in all cases have been shown in the Figs. of ref.[4].

that the model could only lead to uninteresting small neutrino masses of order $m_{\nu_i} \simeq (m_{u_i}/M_G)^2 M_W \leq 10^{-17}$ eV (m_{u_i} being the mass of the up-quark of the i-th generation), because of the so-called "super-suppression" seesaw mechanism, as a resulting of the mixing at the GUT mass scale of the RH neutrinos with the singlets ϕ_α. Then, it was suggested [7] that some of the neutrinos could get phenomenologically more interesting masses given by $m_{\nu_i} \simeq m_{u_i}^2/M_R \sim 10^{-4}$ eV to 100 keV, as a consequence of large Majorana masses for the RH neutrinos of order $M_R \simeq 10^8$ GeV, induced radiatively at the 2-loop level via the so-called Witten diagram [8]. Nevertheless, as was remarked in ref.[3], such a mechanism cannot be used in the more interesting (from the point of view of the string) supersymmetric case, because of the SUSY protection of the gauge hierarchy which prevents the radiative generation of masses larger than the SUSY breaking scale [9], in the TeV range. On the other hand, it was suggested [3] that even in this SUSY case a large RH Majorana mass scale of order $M_R \simeq M_G^2/M_S \simeq 10^{14}$ GeV (where M_S=10^{18} GeV is the string unification scale) might be induced by introducing suitable non-renormalizable terms, like $(F\bar{H}\bar{H}F)/M_S$, which are allowed but not completely specified by the string theory [10]. In this case the neutrino masses are expected to be in the range 10^{-10} to 0.1 eV, still interesting for example for the solution of the solar neutrino problem à la MSW [11]. Finally, in ref.[4] it was shown that the neutrino mass spectrum could drammatically change in presence of large vacuum expectation values for the RH sneutrinos. In particular, it was suggested that in this way one could get phenomenologically interesting masses without the need of the non-renormalizable terms. However, the model was studied only at a single-generation level, and considering only the limit cases where either the effects of such VEVs, or the usual (*i.e.*, for all $<\nu_i^c>=0$) "super-suppression" seesaw mechanism, were dominant. In the present paper, instead, we shall be able to derive a more general expression for the three-generation light neutrino mass matrix. Interestingly this formula will show that, at least as far as the matrix of the couplings λ_6 is non-singular, the part of the neutrino mass matrix due to the presence of non-vanishing $<\nu_i^c>$ turns out to be independent on the particular structure (in the generation space) of the Yukawa-type couplings entering in the superpotential, depending only on the size of these VEVs and on the up-quark masses. But even more interesting is the unexpected fact that the particular structure of the resulting mass matrix, having a rank equal to 1, may lead to only one neutrino mass proportional

to the $<\nu_i^c>$, independently on the number of these VEVs which are non-zero and on the other arbitrary parameters of the model. In other words, unless all $<\nu_i^c>$ are zero, the model predicts that only one neutrino may have a phenomenologically interesting mass, while the other two are expected to be much lighter, having eventually a tiny mass induced by the super-suppression seesaw mechanism.

Our starting point for the calculation of the effective mass matrix for the ordinary light neutrinos is the full mass matrix for the neutral fermions which has already been introduced in ref.[4]. Its structure may easily be obtained by analyzing the role played by the different terms of the superpotential given in eq.(1). In particular, dropping the piece relative to the massive ($\sim M_G$) state ν_H^c, since it does not affect the light neutrino sector, we may write the full mass matrix in the basis (ν_i, ν_j^c, ϕ_α, $\bar{\nu}_H^c$), with i, j, α= 1,2,3, as:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & U & 0 & 0 \\ U^T & \cdot & G & \cdot \\ 0 & G^T & X & V \\ 0 & \cdot & V^T & M_G \end{pmatrix}, \tag{4}$$

where U is the up-quark mass matrix and $G = \lambda_6 \ <\bar{\nu}_H^c> \equiv \lambda_6 \, M_G$ is the 3×3 matrix describing the mixing between the RH neutrinos and the singlets ϕ_α, assumed to be non-singular. The "dots" stand for entries not larger than the weak scale which do not play an essential role in our discussion. $X = \lambda_8 \ <\phi_o> \simeq \lambda_8 \, M_W$ is the 3×3 mass matrix of the singlet fields ϕ_α. M_G in the lower-right corner of eq.(4) corresponds to the typical mass at the GUT scale for the fermionic partner of the Higgs field $\bar{\nu}_H^c$ after the spontaneous symmetry breaking. The new interesting piece of $\mathcal{M}_\nu$ is the 3×1 submatrix V describing the terms given in eq.(3), which is due to the presence of non-vanishing VEVs for the scalar partners of the RH neutrinos. We notice that it arises from the same term in the superpotential as G, and may be written as:

$$V = (\lambda_6)^T \xi \,,$$

proportional to the transposed-conjugated of λ_6 and to the 3×1 vector $\xi = (<\nu_1^c>, <\nu_2^c>, <\nu_3^c>)^T$. Now we may proceed to the evaluation of the "effective" mass matrix for the three ordinary light neutrinos. Following a usual procedure employed in the seesaw-type of models, we may write:

$$m_\nu \simeq (U \;\; 0 \;\; 0) \begin{pmatrix} \cdot & G & \cdot \\ G^T & X & V \\ \cdot & V^T & M_G \end{pmatrix}^{-1} \begin{pmatrix} U^T \\ 0 \\ 0 \end{pmatrix} \equiv U\,(R^{-1})\,U^T\,, \qquad (5)$$

where, approximately (up to terms of order $< \nu_i^c > /M_G$), the effective 3×3 RH Majorana mass matrix R may be expressed as:

$$R \simeq (G \;\; 0) \begin{pmatrix} X & V \\ V^T & M_G \end{pmatrix}^{-1} \begin{pmatrix} G^T \\ 0 \end{pmatrix} \equiv G\,(\Omega^{-1})\,G^T\,. \qquad (6)$$

Similarly, the effective 3×3 matrix Ω can be obtained by means of the formula:

$$\Omega \simeq X - \frac{1}{M_G}\,V\,V^T\,. \qquad (7)$$

Using eqs.(5-7) we can then re-write our effective mass matrix for the three light neutrinos as:

$$m_\nu \simeq U\,(R^{-1})\,U^T \;\simeq U\,G^{T-1}\,\Omega\,G^{-1}\,U^T \;\simeq (U\,G^{T-1})\,(X - \frac{1}{M_G}\,V\,V^T)\,(G^{-1}\,U^T)\,. \qquad (8)$$

Therefore, using the definitions of G and V given above, we get finally:

$$m_\nu \simeq \frac{1}{M_G^2}\,U\,\left\{ \left(\lambda_6^{T-1}\,X\,\lambda_6^{-1}\right) \,-\, \frac{1}{M_G}\,\left(\xi\,\xi^T\right) \right\}\,U^T\,. \qquad (9)$$

This formula, which is our main result, gives the approximate expression of the neutrino mass matrix in the more general case. As we mentioned above, a remarkable feature is the cancellation of the explicit dependence of the second term on the unknown structure of the Yukawa-type couplings λ_6. This means therefore that we may get predictions for the non-negligibly small neutrino masses, due to the presence of $< \nu_i^c >$, only specifying the pattern of the VEVs for the RH sneutrinos. Of course we are aware that the procedure used above for its derivation may be not quite appropriate, especially for $< \nu_i^c >$ of order M_G, but we have checked through a numerical analysis that it leads to the correct behaviour for the neutrino mass spectrum. Furthermore, at a simple single-generation level, the two terms in eq.(9) lead, respectively, to the formulas already obtained in refs.[3,4] in the limits $< \nu^c > \rightarrow 0$ and $< \nu^c > \gg \sqrt{M_W M_G} \simeq 10^9$ GeV. In fact, the first term corresponds to the "super-suppression" seesaw, which typically gives masses $m_{\nu_i} \simeq (m_{ui}^2/M_G)^2\,M_W \sim 10^{-25}$

to 10^{-17} eV, negligibly too small for any phenomenological application. The second term, on the other hand, is due to the non-vanishing of some of the $<\nu^c>$, and may lead to larger neutrino masses given by $m_{\nu_i} \simeq (m^2_{ui} <\nu^c_i>)^2/M^3_G$. In particular, for $<\nu^c> \simeq M_G$, it gives $m_\nu \simeq m^2_u/M_G$, recovering the result of the standard-GUT seesaw scenarios, with the scale of the RH Majorana masses set at M_G. Nevertheless, a closer view of the structure of eq.(9) shows that, independently on the form assumed for the up-quark mass matrix, only one neutrino may get a mass proportional to $<\nu^c>$. The reason for this strong statement, which implies therefore that in the flipped model at least two neutrinos are expected to have extremely small masses, is due to the structure of the matrix in the second term of eq.(9). In fact, since it is given by the product of a 3×1 vector $(U\,\xi)$ times its transposed-conjugated, it may only have a rank equal to one, corresponding to only one non-zero eigen-mass given by:

$$m_{\nu_3} \simeq \frac{1}{M^3_G}\,|U\,\xi\,|^2\,. \tag{10}$$

In this equation $|\,U\,\xi\,|^2$ is the modulo-squared of the 3×1 vector obtained by the product of the up-quark mass matrix with the vector ξ of the VEVs $<\nu^c_i>$. The other two neutrinos, on the other hand, will in general get a very tiny mass, either through the first term of eq.(9), or still from the second term, but at a higher order of approximation.(and, In other words we predict that, unless all VEVs $<\nu^c_i>$ are set to zero, the light neutrino spectrum in the flipped model is made by two very light states and one heavier neutrino with a mass given by eq.(10). As an example, assuming for simplicity a diagonal up-quark mass matrix, one would get:

$$m_{\nu_3} \simeq \frac{1}{M^3_G}\left\{\left(m_u <\nu^c_1>\right)^2 + \left(m_c <\nu^c_2>\right)^2 + \left(m_t <\nu^c_3>\right)^2\right\}.$$

In particular, for $<\nu^c_3> \simeq M_G$, the third (say, τ-) neutrino mass will be given by $m_{\nu_3} \simeq m^2_t/M_G \simeq 10^{-3}$ eV, in the correct range for solving the solar neutrino problem via the MSW oscillations $\nu_e \to \nu_3$. We wish to stress here, once more, that this result holds in the present model under quite general assumptions, namely the non-singularity of the Yukawa-type matrix λ_6 and the presence of VEVs for the RH sneutrinos at the GUT mass scale.

In conclusion, what we have done in the present paper, is a detailed analysis of the light neutrino mass spectrum which may be obtained in the context of the supersymmetric flipped $SU(5) \otimes U(1)$ model, obtaining a general (approximate) formula for the corresponding three-generation mass matrix. The particular structure of such expression has also suggested that the presence of large ($\simeq M_G$) VEVs for the RH sneutrinos may at most lead to only one neutrino mass in the range of phenomenological interest for astrophysics and cosmology. The other two neutrinos, instead, are expected to have a too small mass.

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